Class IX Session 2023-24
Subject - Mathematics
Sample Question Paper - 10

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. $(1296)^{\frac{-1}{4}}$ = **[1]**

   a) 6 b) $-\frac{1}{6}$

   c) -6 d) $\frac{1}{6}$

2. The graph of y = 6 is a line **[1]**

   a) Parallel to x-axis at a distance 6 units from the origin b) Making an intercept 6 on the x- axis.

   c) Making an intercept 6 on both the axes. d) Parallel to y-axis at a distance 6 units from the origin

3. Signs of the abscissa and ordinate of a point in the second quadrant are respectively. **[1]**

   a) (-, +) b) (+, -)

   c) (+, +) d) (-, -)

4. In a histogram the area of each rectangle is proportional to **[1]**

   a) the class size of the corresponding class interval b) cumulative frequency of the corresponding class interval

   c) the class mark of the corresponding class interval d) frequency of the corresponding class interval

5. If (2, 0) is a solution of the linear equation 2x + 3y = k, then the value of k is **[1]**

a) 2 b) 4

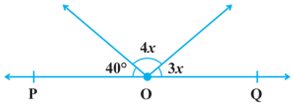
c) 5 d) 6

6. The things which are double of the same things are **[1]**

a) equal b) halves of the same thing

c) unequal d) double of the same thing

7. In the figure, POQ is a line. The value of x is **[1]**



a) 25° b) 30°

c) 20° d) 35°

8. If a diagonal AC and BD of a quadrilateral ABCD bisect each other, then ABCD is a **[1]**

a) Parallelogram b) Rhombus

c) Rectangle d) Triangle

9. If $10x - 4x^2 - 3$, then the value of $p(0) + p(1)$ is **[1]**

a) -3 b) 0

c) 3 d) 1

10. **The cost of a notebook is twice the cost of a pen.** The equation to represent this statement is **[1]**

a) 2x = 3y b) x = 3y

c) none of these d) x - 2y = 0

11. In $\triangle$ABC, E is the mid-point of median AD such that BE produced meets AC at F. If AC = 10.5 cm, then AF = **[1]**

a) 2.5 cm b) 5 cm

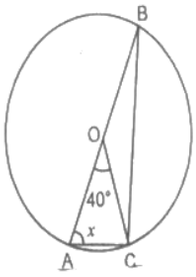
c) 3 cm d) 3.5 cm

12. D and E are the mid-points of the sides AB and AC of $\triangle$ABC and O is any point on the side BC, O is joined to A. If P and Q are the mid-points of OB and OC res, Then DEQP is **[1]**

a) A Triangle b) A Rectangle

c) A Rhombus d) A Parallelogram

13. In a figure, O is the centre of the circle with AB as diameter. If $\angle AOC = 40^o$, the value of x is equal to **[1]**



a) 80^o b) 50^o

c) 70^o d) 60^o

14. The value of 1.9999……. in the form $\frac{p}{q}$, where 'p' and 'q' are integers and q $\neq$ 0, is **[1]**

a) $\frac{1999}{1000}$ b) $\frac{19}{10}$

c) 2 d) $\frac{1}{9}$

15. The force applied on a body is directly proportional to the acceleration produced on it. The equation to represent the above statement is **[1]**

a) y = kx b) y = x

c) y + x = 0 d) none of these

16. In a $\triangle ABC$, if $3\angle A = 4\angle B = 6\angle C$ then A : B : C = ? **[1]**

a) 6 : 4 : 3 b) 2 : 3 : 4

c) 3 : 4 : 6 d) 4 : 3 : 2

17. To draw a histogram to represent the following frequency distribution : **[1]**

Class interval	5-10	10-15	15-25	25-45	45-75
Frequency	6	12	10	8	15

The adjusted frequency for the class 25-45 is

a) 6 b) 5

c) 2 d) 3

18. To make a closed hollow cone of base radius 7 cm and height 24 cm, the area of metal sheet required is **[1]**

a) 704 cm^2 b) 825 cm^2

c) 1100 cm^2 d) 550 cm^2

19. **Assertion (A):** The sides of a triangle are 3 cm, 4 cm and 5 cm. Its area is 6 cm^2. **[1]**

Reason (R): If 2s = (a + b + c), where a, b, c are the sides of a triangle, then area = $\sqrt{(s-a)(s-b)(s-c)}$.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

20. **Assertion (A):** The equation of 2x + 5 = 0 and 3x + y = 5 both have degree 1. **[1]**

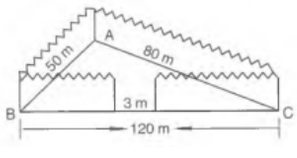
Reason (R): The degree of a linear equation in two variables is 2.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

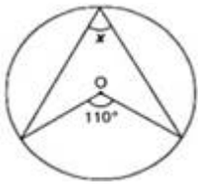
c) A is true but R is false. d) A is false but R is true.

Section B

21. A triangular park ABC has sides 120 m, 80 m and 50 m (in a given figure). A gardener Dhania has to put a fence all around it and also plant grass inside. How much area does she need to plant? Find the cost of fencing it with barbed wire at the rate of ₹ 20 per metre leaving a space 3m wide for a gate on one side. **[2]**

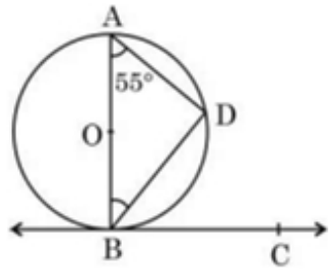


22. Find an angle marked as x in given figure where O is the centre of the circle: **[2]**



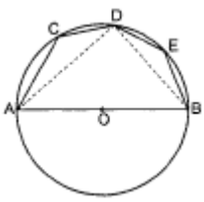
23. A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of ₹32 per 100 cm^2. **[2]**

24. In Fig., BC is tangent to the circle at point B of circle centred at O. BD is a chord of the circle so that ∠BAD = 55^o. Find m ∠DBC. **[2]**



OR

In given figure, AOB is a diameter of the circle and C, D, E are any three points on the semi-circle. Find the value of ∠ACD + ∠ BED.



25. Express the linear equation in the form ax + by + c = 0 and indicate the values of a, b and c in $2x + 3y = 9.3\overline{5}$ **[2]**

OR

Find whether(4, 0) is the solution of the equation x – 2y = 4 or not?

Section C

26. Solve the equation for x: $3^{2x+4} + 1 = 2 \times 3^{x+2}$ **[3]**

27. Find the value of the polynomial $3x^3 - 4x^2 + 7x - 5$, when x = 3 and also when x = -3. **[3]**

28. The perimeter of an isosceles triangle is 32 cm. The ratio of the equal side to its base is 3: 2. Find the area of the triangle. **[3]**

OR

Find the cost of laying grass in a triangular field of sides 50 m, 65 m and 65 m at the rate of Rs7 per m^2.

29. Find the solution of the linear equation x + 2y = 8 which represents a point on **[3]**

i. The x-axis

ii. The y-axis

30. P is the mid-point of the side CD of a parallelogram ABCD. A line through C parallel to PA intersects AB at Q and DA produced at R. Prove that DA = AR and CQ = QR. **[3]**

OR

Show that the quadrilateral formed by joining the mid-points the sides of a rhombus, taken in order, form a rectangle.

31. Write the answer of each of the following questions: **[3]**

i. What is the name of horizontal and the vertical lines drawn to determine the position of any point in the Cartesian plane?

ii. What is the name of each part of the plane formed by these two lines?

iii. Write the name of the point where these two lines intersect.

Section D

32. Find the values of a and b if $\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} = a + b\sqrt{5}$. **[5]**

OR

If $a = \frac{\sqrt{2}+1}{\sqrt{2}-1}$ and $b = \frac{\sqrt{2}-1}{\sqrt{2}+1}$, then find the value of $a^2 + b^2 - 4ab$.

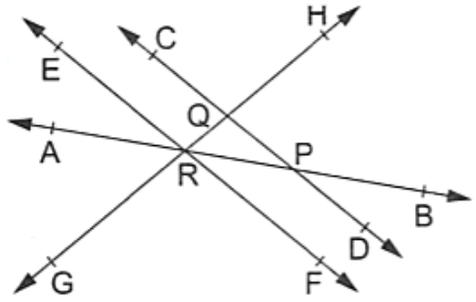
33. In the adjoining figure, name: **[5]**

i. Two pairs of intersecting lines and their corresponding points of intersection

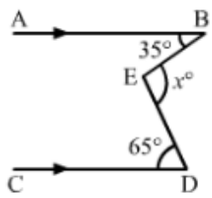
ii. Three concurrent lines and their points of intersection

iii. Three rays

iv. Two line segments

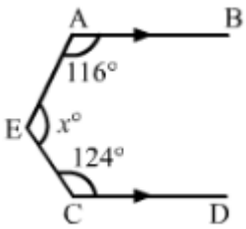


34. In each of the figures given below, AB || CD. Find the value of x° in each case. **[5]**



OR

In each of the figures given below, AB || CD. Find the value of x° in each other case.



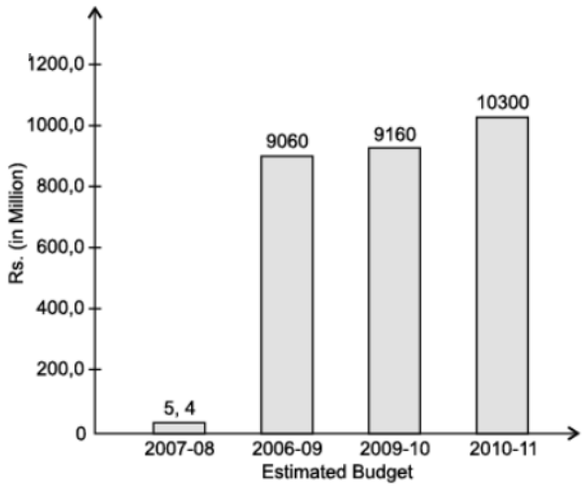
35. What must be added to $x^4 + 2x^3 - 2x^2 + x - 1$ so that the result is exactly divisible by $x^2 + 2x - 3$ **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Ladli Scheme was launched by the Delhi Government in the year 2008. This scheme helps to make women strong and will empower a girl child. This scheme was started in 2008.

The expenses for the scheme are plotted in the following bar chart.



(i) What are the total expenses from 2009 to 2011?

(ii) What is the percentage of no of expenses in 2009-10 over the expenses in 2010-11?

(iii) What is the percentage of minimum expenses over the maximum expenses in the period 2007-2011?

OR

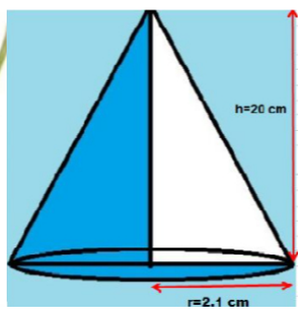
What is the difference of expenses in 2010-11 and the expenses in 2006-09?

37. **Read the text carefully and answer the questions:** **[4]**

Once upon a time in Ghaziabad was a corn cob seller. During the lockdown period in the year 2020, his business was almost lost.

So, he started selling corn grains online through Amazon and Flipcart. Just to understand how many grains he will have from one corn cob, he started counting them.

Being a student of mathematics let's calculate it mathematically. Let's assume that one corn cob (see Fig.), shaped somewhat like a cone, has the radius of its broadest end as 2.1 cm and length as 20 cm.



(i) Find the curved surface area of the corn cub.

(ii) What is the volume of the corn cub?

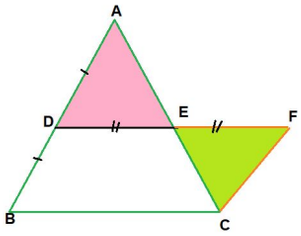
(iii) If each 1 cm^2 of the surface of the cob carries an average of four grains, find how many grains you would find on the entire cob?

OR

How many such cubs can be stored in a cartoon of size 20 cm $\times$ 25 cm $\times$ 20 cm.

38. **Read the text carefully and answer the questions:** **[4]**

Haresh and Deep were trying to prove a theorem. For this they did the following



i. Draw a triangle ABC

ii. D and E are found as the mid points of AB and AC

iii. DE was joined and DE was extended to F so DE = EF

iv. FC was joined.

(i) $\triangle ADE$ and $\triangle EFC$ are congruent by which criteria?

(ii) Show that $CF \parallel AB$.

(iii) Show that CF = BD.

OR

Show that DF = BC and $DF \parallel BC$.

Solution

Section A

1.

(d) $\frac{1}{6}$

Explanation: $(1296)^{\frac{-1}{4}} = \left(6^4\right)^{\frac{-1}{4}}$

$= (6)^{-1} = \frac{1}{6}$

2. **(a)** Parallel to x-axis at a distance 6 units from the origin

Explanation: As y = a is an equation of a line parallel to x-axis at a distance of a units from the origin.

3. **(a)** (-, +)

Explanation: Abscissa means just the horizontal axis i.e. x axis. Ordinate means just the vertical axis i.e. y axis. x-corrdinate and y-corrdinate make a point.
The signs of abscissa and ordinate of a point in quadrant II are (-, +).

4.

(d) frequency of the corresponding class interval

Explanation: A histogram is a display of statistical information that uses rectangles to show the frequency of data items in successive numerical intervals of equal size. In the most common form of histogram, the independent variable is plotted along the horizontal axis and the dependent variable is plotted along the vertical axis.

5.

(b) 4

Explanation: (2, 0) is a solution of the linear equation 2x + 3y = k

$\Rightarrow 4 = k$

6. **(a)** equal

Explanation: According to an Euclidian axiom, The things which are double of the same things are equal to one another.
Example : if 2x = 2y, then x = y.

7.

(c) 20°

Explanation: Since, POQ is a line segment.

$\therefore \angle$POQ = 180^o

$\Rightarrow \angle POA + \angle AOB + \angle BOQ = 180^\circ$

$\Rightarrow 40^o + 4x + 3x = 180^o$

[Putting $\angle POA = 40^\circ, \angle AOB = 4x$ and $\angle BOQ = 3x$]

$\Rightarrow 7x = 180^o - 40^o$

$\Rightarrow 7x = 140^o$

$\Rightarrow x = \frac{140^\circ}{7}$

$\therefore x = 20^o$

8. **(a)** Parallelogram

Explanation: Two diagonals of quadrilateral form four triangles. Out of these four triangles two triangles of opposite to each other are congruent by SAS. By using CPCT property we can prove that both pair of opposite sides in a quadrilateral are parallel. A quadrilateral with both pair of opposite sides parallel is called parallelogram.

9.

(b) 0

Explanation: $10x - 4x^2 - 3$

$p(x) = -4x^2 + 10x - 3$

=> $p(0) + p(1) = \left[-4(0)^2 + 10(0) - 3\right] + \left[-4(1)^2 + 10(1) - 3\right]$

=> $p(0) + p(1) = [0 + 0 - 3] + [-4 + 10 - 3]$
=> $p(0) + p(1) = [-3] + [3]$
=> $p(0) + p(1) = 0$

10.

(d) x - 2y = 0

Explanation: Let the cost of the notebook is ₹ x and pen is ₹ y and we have given that the cost of a notebook is twice the cost of a pen.

So we have

x = 2y

or x - 2y = 0

11.

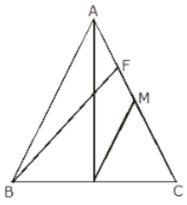
(d) 3.5 cm

Explanation:

Given,

In △ABC

E is mid point of median AD



AC = 10.5 cm

Draw DM || EF

∵ E is mid point of AD so F is mid point of AM

AF = FM ...(i)

In △BFC

EF || DM

So, FM = MC ...(ii)

From (i) & (ii)

AF = MC ...(iii)

AC = AF + MC + FM

⇒ AC = AF + AF + AF From (i) (ii) &(iii)

AC = 3AF

AF = $\frac{1}{3}$ AC

AF = $\frac{1}{3} \times 10.5 = 3.5$ cm

12.

(d) A Parallelogram

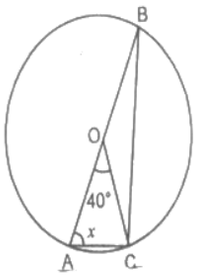
Explanation: By mid point theorem, DE is parallel to BC. In triangle BOA, DP parallel to OA and OA is parallel to QE in triangle AOC(mid point theorem) because D and P are mid points in triangle BOA and E and Q are mid points in triangle AOC.

So, DP is parallel to EQ. In quadrilateral DPQE, both pair of opposite sides are parallel. So, it become parallelogram.

13.

(c) 70^o

Explanation:



OA = OC (radii)
So, ∠OAC = ∠OCA = x
Again, In △OAC

$\angle OAC + \angle OCA + \angle AOC = 180^o$

$x + x + \angle AOC = 180^o$

$x + x + 40^o = 180^o$

$2x = 140^o$

$x = 70^o$

14.

(c) 2
Explanation: 1.9999 can be written as 2,
2is taken as approx vlaue .

15. **(a)** y = kx
Explanation: let force applied be y and accleration produced be x
The force applied on a body is directly proportional to the acceleration produced on it.

$y \alpha x$
$y = kx$
where k is proportionality constant

16.

(d) 4 : 3 : 2
Explanation:
3A = 4B = 6C
3A = 4B ⇒ A/B = 4/3
This means, A:B=4:3
4B = 6C ⇒ B/C = 6/4 = 3/2
⇒ B:C = 3:2
The value of B in 4:3 is equal to the value of B in 3:2
Hence A : B : C = 4 : 3 : 2

17.

(c) 2
Explanation: Adjusted frequency = $\left(\frac{\text{frequency of the class}}{\text{width of the class}}\right) \times 5$
Therefore, Adjusted frequency of 25 - 45 = $\frac{8}{20} \times 5 = 2$

18. **(a)** 704 cm^2
Explanation: Given r = 7 cm, h = 24 cm
So, slant height, l = $\sqrt{r^2 + h^2}$
= $\sqrt{7^2 + 24^2}$
= $\sqrt{49 + 576}$
= $\sqrt{625}$
= 25 cm

So, surface area of hollow cone = curved surface area + area of base

$= \pi rl + \pi r^2$

$= \pi r(l + r)$

$= \frac{22}{7} \times 7\ (25 + 7)$

$= 22 \times 32$

$= 704\ cm^2$

19.

(c) A is true but R is false.

Explanation: $s = \frac{a+b+c}{2}$

$s = \frac{3+4+5}{2} = 6$ cm

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(6)(6-3)(6-4)(6-5)}$

$= \sqrt{(6)(3)(2)(1)} = 6\ cm^2$

20.

(c) A is true but R is false.

Explanation: Every linear equation has degree 1.

2x + 5 = 0 and 3x + y = 5 are linear equations. So, both have degree 1.

Section B

21. Computation of area: Clearly, the park is trianglar with sides

a = BC = 120 m, b = CA = 80 m and c = AB = 50 m

Ifs denotes the semi-perimeter of the park, then

$2s = a + b + c \Rightarrow 2s = 120 + 80 + 50 \Rightarrow s = 125$

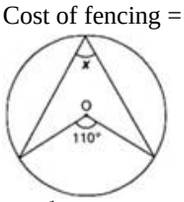
$\therefore$ s - a = 125 - 120 = 5, s - b = 125 - 80 = 45 and s - c = 125 - 50 = 75

Hence, Area of the park $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{125 \times 5 \times 45 \times 75}m^2 = 375\sqrt{15}\ m^2$

Length of the wire needed for fencing = perimeter of the park - width of the gate

= 250m - 3m = 247 m

Cost of fencing = Rs.(20×247) = Rs.4940

22. 

$x = \frac{1}{2} \times 110° = 55°$

[$\because$ Angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point of the remaining part of the circle.]

23. Inner radius of the bowl, $r = \frac{10.5}{2} = 5.25$ cm

Therefore, Inner curved surface area of the bowl $= 2\pi r^2 = 2 \times \frac{22}{7} \times (5.25)^2 = 173.25\ cm^2$

Rate of tin-plating = 32 per 100 cm^2

Therefore, Cost of tin-plating the bowl on the inside

= Inner curved surface area of the bowl $\times$ Rate of tin-plating

$= 173.25 \times \frac{32}{100}$

= 55.44

Therefore, the cost of tin-plating the bowl on the inside is 55.44.

24. In $\triangle$ADB,

$\angle ADB = 90^o$ (Angle in semi-circle) ...(i)

Now, by using angle sum properly in $\triangle$ABD, we have

$\angle ABD + \angle ADB + \angle DAB = 180^o$

$\Rightarrow \angle ABD + 90^o + 55^o = 180^o$ ...[From equation (i) and given $\angle DAB = 55^o$]

$\Rightarrow \angle ABD + 180o - 145^o$

$\Rightarrow \angle ABD = 35o$ ...(ii)

Now, ∠ABD = 90^o ...(Angle between tangent and radius)

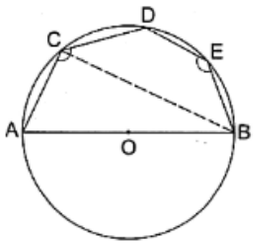
or, ∠ABD + ∠DBC = 90^o

or, ∠DBC = 90^o - ∠ABD

or, ∠DBC = 90^o - 35^o ...[From equation (ii)]

or, ∠DBC = 55^o

OR



Join BC,

Then, ∠ACB = 90° (Angle in the semicircle)

Since DCBE is a cyclic quadrilateral.

∠BCD + ∠BED = 180°

Adding ∠ACB both the sides, we get

∠BCD + ∠BED + ∠ACB = ∠ACB +180°

(∠BCD + ∠ACB) + ∠BED = 90° + 180°

∠ACD + ∠BED = 270°

25. $2x + 3y = 9.3\bar{5}$

We need to express the linear equation $2x + 3y = 9.3\bar{5}$ in the form ax + by + c = 0 and indicate the values of a, b and c.

$2x + 3y = 9.3\bar{5}$ can also be written as $2x + 3y - 9.3\bar{5} = 0$.

We need to compare the equation $2x + 3y - 9.3\bar{5} = 0$

with the general equation ax + by + c = 0, to get the values of a, b and c.

Therefore, we can conclude that $a = 2, b = 3$ and $c = -9.3\bar{5}$

OR

x-2y=4

Put x = 4 and y = 0 in given equation, we get

x – 2y = 4 – 2(0) = 4

∴ (4, 0) is a solution of given equation.

Section C

26. $3^{2x+4} + 1 = 2 \times 3^{x+2}$

$\Rightarrow 3^{2x} \cdot 3^4 + 1 = 2 \cdot 3^x \cdot 3^2$

$\Rightarrow (3^x)^2 \times 81 + 1 = 18.3^x$

$\Rightarrow 81(3^x)^2 - 18.3^x + 1 = 0$

Let $3^x = a$

Thus, we have $81a^2 - 18a + 1 = 0$

$\Rightarrow 81a^2 - 9a - 9a + 1 = 0$

$\Rightarrow (9a)^2 - 2(9a)(1) + (1)^2 = 0$

$\Rightarrow (9a - 1)^2 = 0$

$\Rightarrow 9a - 1 = 0$

$\Rightarrow 9a = 1$

$\Rightarrow a = \frac{1}{9}$

$\Rightarrow 3^x = \frac{1}{9}$

$\Rightarrow 3^x = \frac{1}{3^2} = 3^{-2}$

$\Rightarrow x = -2$

27. Let $p(x) = 3x^3 - 4x^2 + 7x - 5$

$\therefore p(3) = 3(3)^3 - 4(3)^2 + 7(3) - 5$

= 3(27) - 4(9) + 21 - 5

= 81 - 36 + 21 - 5

= 61

Now, p(-3) = $3(-3)^3 - 4(-3)^2 + 7(-3) - 5$

= 3(-27) - 4(9) - 21 - 5

= - 81 - 36 - 21 - 5

= -143

28. As the sides of the equal to the base of an isosceles triangle is 3 : 2, so let the sides of an isosceles triangle be 3x, 3x and 2x.

Now, perimeter of triangle = 3x + 3x + 2x = 8x

Given Perimeter of triangle = 32 m

$\therefore 8x = 32; x = 32 \div 8 = 4$

So, the sides of the isosceles triangle are $(3 \times 4)cm, (3 \times 4)cm, (2 \times 4)cm$ i.e., 12 cm, 12 cm and 8cm

$\therefore\ s = \frac{12+12+8}{2} = \frac{32}{2} = 16cm$

$= \sqrt{16(16-12)(16-12)(16-8)}$

$= \sqrt{16 \times 4 \times 4 \times 8} = \sqrt{4 \times 4 \times 4 \times 4 \times 4 \times 2}$

$= 4 \times 4 \times 2\sqrt{2} = 32\sqrt{2}cm^2$

OR

We have, 2s = 50 m + 65 m + 65 m = 180 m

S = 180 ÷ 2 = 90 m

Area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{90(90-50)(90-65)(90-65)}$

$= \sqrt{90 \times 40 \times 25 \times 25} = 60 \times 25$

= 1500m^2.

Cost of laying grass at the rate of Rs7 per m^2 = Rs(1500 × 7) = Rs10,500.

29. i. On x-axis y = 0

$\Rightarrow\ x + 2 \times 0 = 8 \Rightarrow\ x = \ 8$

Therefore, the required point is (8, 0).

ii. On y-axis x = 0

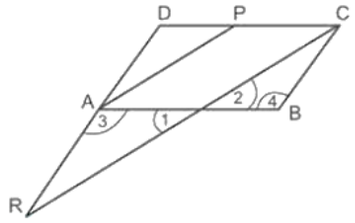
$\Rightarrow\ 0 + 2y = 8$

$\Rightarrow\ y = \frac{8}{2} \Rightarrow\ y = 4$

Thus, the required point is (0, 4).

30. ABCD is a parallelogram. P is the mid-point of CD. CR which intersects AB at Q is parallel to AP

In $\triangle DCR$, P is the mid-point of CD and AP || CR,



$\therefore$ A is the mid-point of DR, i.e., AD = AR.

[$\because$The line drawn through the mid-point of one side of a triangle parallel to another side intersects the third side at its mid-point.]

In $\triangle ARQ$ and $\triangle BCQ$, we have

AR = BC [$\because$ AD = AR [proved above) and AD = BC]

$\angle 1 = \angle 2$ [Vertically opposite angles]

$\angle 3 = \angle 4$ [Alt. $\angle s$]

$\therefore \triangle ARQ \cong \triangle BCQ$ [By AAS Congruence rule]

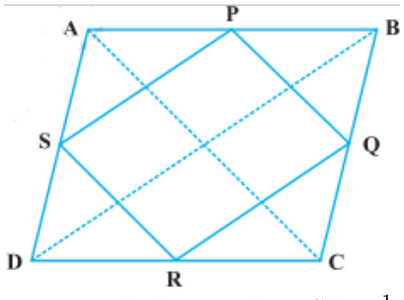
CQ = QR [CPCT]

CQ = QR

Hence, DA = AR and CQ = QR

OR

Let ABCD be a rhombus and P, Q, R and S be the mid-points of sides AB, BC, CD and DA, respectively (Fig.). Join AC and BD.



From triangle ABD, we have $SP = \frac{1}{2}BD$ and

SP || BD (Because S and P are mid-points)

Similarly $RQ = \frac{1}{2}BD$ and RQ || BD

Therefore, SP = RQ and SP || RQ

So, PQRS is a parallelogram ...(1)

Also, AC $\perp$ BD (Diagonals of a rhombus are perpendicular)

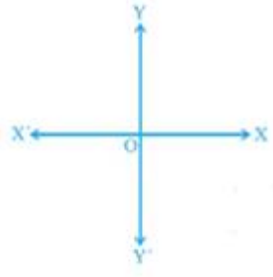
Further PQ || AC (From $\triangle$BAC)

As SP || BD, PQ || AC and AC $\perp$ BD,

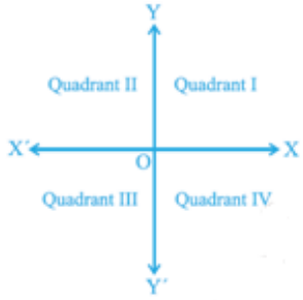
therefore, we have BD $\perp$ PQ, i.e. $\angle$SPQ = 90º. ..(2)

Therefore, PQRS is a rectangle. [From (1) and (2)]

31. i. The horizontal line that is drawn to determine the position of any point in the Cartesian plane is called as x-axis. The vertical line that is drawn to determine the position of any point in the Cartesian plane is called as y-axis



ii. The name of each part of the plane that is formed by x-axis and y-axis is called as quadrant.



iii. The point, where the x-axis and the y-axis intersect is called as origin.

Section D

32. LHS

$= \frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$

$= \frac{7+3\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$

$= \frac{7\times3-7\sqrt{5}+3\sqrt{5}\times3-3\sqrt{5}\times\sqrt{5}}{3^2-\sqrt{5}^2} - \frac{7\times3+7\sqrt{5}-3\sqrt{5}\times3-3\sqrt{5}\times\sqrt{5}}{3^2-\sqrt{5}^2}$

$= \frac{21-7\sqrt{5}+9\sqrt{5}-15}{9-5} - \frac{21+7\sqrt{5}-9\sqrt{5}-15}{9-5}$

$= \frac{6+2\sqrt{5}}{4} - \frac{6-2\sqrt{5}}{4}$

$= \frac{6+2\sqrt{5}-6+2\sqrt{5}}{4}$

$= \frac{0+4\sqrt{5}}{4}$

$= 0 + \sqrt{5}$

We know that,

$\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} = a + b\sqrt{5}$

$0 + \sqrt{5} = a + b\sqrt{5}$

a = 0 and b = 1

OR

Given, $a = \frac{\sqrt{2}+1}{\sqrt{2}-1}$ and $b = \frac{\sqrt{2}-1}{\sqrt{2}+1}$

Here, $a = \frac{\sqrt{2}+1}{\sqrt{2}-1} = \frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{(\sqrt{2}+1)^2}{(\sqrt{2})^2-1^2}$

$= \frac{(\sqrt{2})^2+1+2\sqrt{2}}{2-1} = \frac{2+1+2\sqrt{2}}{1} = 3 + 2\sqrt{2}$

$\therefore \quad a = 3 + 2\sqrt{2}$ ...(i)

$b = \frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2-1}$

$= \frac{(\sqrt{2})^2+1^2-2\sqrt{2}}{2-1} = \frac{2+1-2\sqrt{2}}{1} = 3 - 2\sqrt{2}$

$\therefore \quad b = 3 - 2\sqrt{2}$ ...(ii)

From equation (i) and (ii)

$a + b = 3 + 2\sqrt{2} + 3 - 2\sqrt{2} = 6$

ab = $(3 + 2\sqrt{2})(3 - 2\sqrt{2}) = 3^2 - (2\sqrt{2})^2$

$= 9 - 4 \times 2 = 9 - 8 = 1$

$\therefore a^2 + b^2 - 4ab = a^2 + b^2 + 2ab - 6ab$

$= (a + b)^2 - 6ab$

$= 6^2 - 6$

$= 36 - 6 = 30$

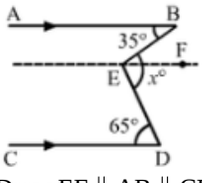
33. i. $\overleftrightarrow{EF}$, $\overleftrightarrow{GH}$ and their corresponding point of intersection is R.

$\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ and their corresponding point of intersection is P.

ii. $\overleftrightarrow{AB}$, $\overleftrightarrow{EF}$, $\overleftrightarrow{GH}$ and their point of intersection is R.

iii. Three rays are: $\overrightarrow{RB}$, $\overrightarrow{RH}$, $\overrightarrow{RG}$

iv. Two line segments are: $\overline{RQ}$, $\overline{RP}$.

34. 

Draw EF || AB || CD

Now, AB || EF and BE is the transversal.

Then,

$\angle ABE = \angle BEF$ [Alternate Interior Angles]

$\Rightarrow \angle BEF = 35^\circ$

Again, EF || CD and DE is the transversal

Then,

$\angle DEF = \angle FED$

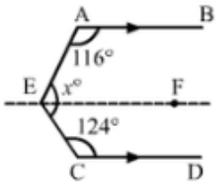
$\Rightarrow \angle FED = 65^\circ$

$\therefore x^\circ = \angle BEF + \angle FED$

$x^\circ = 35^\circ + 65^\circ$

$x^\circ = 100^\circ$

OR



Draw EF || AB || CD

Then, $\angle AEF + \angle CEF = x^\circ$

Now, EF || AB and AE is the transversal

$\therefore \angle AEF + \angle BAE = 180^\circ$ [Consecutive Interior Angles]

$\Rightarrow \angle AEF + 116 = 180$

$\Rightarrow \angle AEF = 64^\circ$

Again, EF $\parallel$ CD and CE is the transversal.

$\angle CEF + \angle ECD = 180^\circ$ [Consecutive Interior Angles]

$\Rightarrow \angle CEF + 124 = 180$

$\Rightarrow \angle CEF = 56^\circ$

Therefore,

$x^\circ = \angle AEF + \angle CEF$

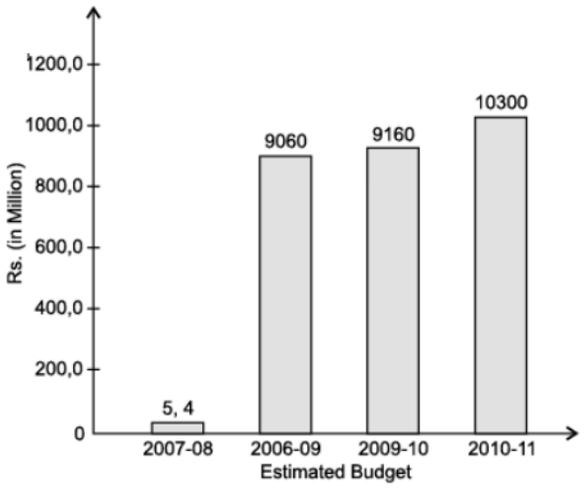
$x^\circ = (64 + 56)^\circ$

$x^\circ = 120^\circ$

35. Let $p(x) = x^4 + 2x^3 - 2x^2 + x - 1$ and $q(x) = x^2 + 2x - 3$. When p(x) is divided by q(x), the remainder is a linear expression in x.
Therefore, let r(x) = ax + b ibe added to p(x) so that p(x) + r(x) is divisible by q(x).
Let
f(x) = p(x) + r(x). Therefore,we have,
$f(x) = x^4 + 2x^3 - 2x^2 + x - 1 + ax + b$
$f(x) = x^4 + 2x^3 - 2x^2 + (a + 1)x + b - 1$
We have, $q(x) = x^2 + 2x - 3 = x^2 + 3x - x - 3 = x(x + 3) - (x + 3) = (x - 1)(x + 3)$
Clearly, q(x) is divisible by (x - 1) and (x + 3) i.e. x - 1 and x + 3 are factors of q(x).
i.e., f(1) = 0 and f(-3) = 0
$\Rightarrow 1 + 2 - 2 + a + 1 + b - 1 = 0$
and, $(-3)^4 + 2(-3)^3 - 2(-3)^2 + (a + 1) \times (-3) + b - 1 = 0$
$\Rightarrow a + b + 1 = 0$ and $81 - 54 - 18 - 3a - 3 + b - 1 = 0$
$\Rightarrow a + b + 1 = 0$ and $-3a + b + 5 = 0$
$\Rightarrow a + b = -1$ and $3a - b = 5$
Therefore,on adding these two equations, we have,
$\Rightarrow (a + b) + (3a - b) = -1 + 5 \Rightarrow 4a = 4 \Rightarrow a = 1$
Putting a = 1 in a + b = - 1, we have,
$1 + b = -1 \Rightarrow b = -2$
$\therefore r(x) = ax + b \Rightarrow r(x) = x - 2$
Therefore, $x^4 + 2x^3 - 2x^2 + x - 1$ will be divisible by $x^2 + 2x - 3$, if x - 2 is added to it.

Section E

36. **Read the text carefully and answer the questions:**
Ladli Scheme was launched by the Delhi Government in the year 2008. This scheme helps to make women strong and will empower a girl child. This scheme was started in 2008.
The expenses for the scheme are plotted in the following bar chart.



(i) Expenses in 2009-10 = 9160 Million

Expenses in 2010-11 = 10300 Million

Total expenses from 2009 to 2011

= 9160 + 10300

= 19460 Million

(ii) Expenses in 2009-10 = 9160 Million

Expenses in 2010-11 = 10300 Million

Thus percentage of no of expenses in 2009-10 over the expenses in 2010-11

$= \frac{9160}{10300} \times 100$

= 88.93%

(iii) The minimum expenses (in 2007-08) = 5.4 Million

The maximum expenses (in 2010-11) = 10300 Million

Thus percentage of no of minimum expenses over the maximum expenses

$= \frac{5.4}{10300} \times 100$

= 0.052%

OR

The expenses in 2010-11 = 10300 Million

The expenses in 2006-09 = 9060 Million

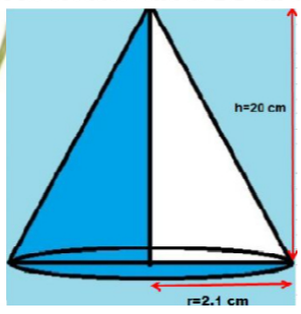
The difference = 10300 - 9060 Million

= 1240 Million

37. **Read the text carefully and answer the questions:**

Once upon a time in Ghaziabad was a corn cob seller. During the lockdown period in the year 2020, his business was almost lost. So, he started selling corn grains online through Amazon and Flipcart. Just to understand how many grains he will have from one corn cob, he started counting them.

Being a student of mathematics let's calculate it mathematically. Let's assume that one corn cob (see Fig.), shaped somewhat like a cone, has the radius of its broadest end as 2.1 cm and length as 20 cm.



(i) First we will find the curved surface area of the corn cob.

We have, r = 2.1 and h = 20

Let l be the slant height of the conical corn cob. Then,

l = $\sqrt{r^2 + h^2} = \sqrt{(2.1)^2 + (20)^2}$ = $\sqrt{4.41 + 400} = \sqrt{404.41}$ = 20.11 cm

$\therefore$ Curved surface area of the corn cub = πrl

= $\frac{22}{7} \times$ 2.1 $\times$ 20.11 cm^2

= 132.726 cm^2 = 132.73 cm^2

(ii) The volume of the corn cub

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 20$

= 92.4 cm^3

(iii) Now

Total number of grains on the corn cob = Curved surface area of the corn cob $\times$ Number of grains of corn on 1 cm^2

Hence, Total number of grains on the corn cob = 132.73 $\times$ 4 = 530.92

So, there would be approximately 531 grains of corn on the cob.

OR

Volume of a corn cub = 92.4 cm^3

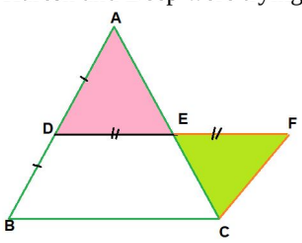
Volume of the cartoon = 20 $\times$ 25 $\times$ 20 = 10,000 cm^3

Thus no. of cubs which can be stored in the cartoon

$\frac{10000}{92.4} \approx 108$ cubs

38. **Read the text carefully and answer the questions:**

Haresh and Deep were trying to prove a theorem. For this they did the following



i. Draw a triangle ABC

ii. D and E are found as the mid points of AB and AC

iii. DE was joined and DE was extended to F so DE = EF

iv. FC was joined.

(i) $\triangle$ADE and $\triangle$CFE

DE = EF (By construction)

$\angle$AED = $\angle$CEF (Vertically opposite angles)

AE = EC(By construction)

By SAS criteria $\triangle$ADE $\cong \triangle$CFE

(ii) $\triangle$ADE $\cong$ $\triangle$CFE

Corresponding part of congruent triangle are equal

$\angle$EFC = $\angle$EDA

alternate interior angles are equal

$\Rightarrow$ AD $\parallel$ FC

$\Rightarrow$ CF $\parallel$ AB

(iii) $\triangle$ADE $\cong$ $\triangle$CFE

Corresponding part of congruent triangle are equal.

CF = AD

We know that D is mid point AB

$\Rightarrow$ AD = BD

$\Rightarrow$ CF = BD

OR

DE = $\frac{BC}{2}$ {line drawn from mid points of 2 sides of $\triangle$ is parallel and half of third side}

DE $\parallel$ BC and DF $\parallel$ BC

DF = DE + EF

$\Rightarrow$ DF = 2DE(BE = EF)

$\Rightarrow$ DF = BC